

## Exercise 14

Given that  $\lim_{x \rightarrow 2}(5x - 7) = 3$ , illustrate Definition 2 by finding values of  $\delta$  that correspond to  $\varepsilon = 0.1$ ,  $\varepsilon = 0.05$ , and  $\varepsilon = 0.01$ .

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### Solution

According to Definition 2, this limit is equivalent to

$$\text{if } |x - 2| < \delta \quad \text{then} \quad |(5x - 7) - 3| < \varepsilon$$

for all positive  $\varepsilon$ . We're looking for a number  $\delta$  that's greater than  $|x - 2|$ .

$$|(5x - 7) - 3| < \varepsilon$$

$$|5x - 10| < \varepsilon$$

$$|5(x - 2)| < \varepsilon$$

$$5|x - 2| < \varepsilon$$

$$|x - 2| < \frac{\varepsilon}{5}$$

If  $\varepsilon = 0.1$ , then choose

$$\delta = \frac{\varepsilon}{5} = \frac{0.1}{5} = 0.02.$$

If  $\varepsilon = 0.05$ , then choose

$$\delta = \frac{\varepsilon}{5} = \frac{0.05}{5} = 0.01.$$

If  $\varepsilon = 0.01$ , then choose

$$\delta = \frac{\varepsilon}{5} = \frac{0.01}{5} = 0.002.$$